

# CANCELLATION OF NONLINEAR INTERSYMBOL INTERFERENCE IN VOICEBAND COMMUNICATION CHANNELS<sup>1</sup>

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## ABSTRACT

A nonlinear cancellation technique for nonlinear voiceband channels is investigated. The technique uses a fast orthogonal search (FOS) algorithm to construct the cancellation filter by choosing terms from a large selection of candidates that have been selected after a theoretical investigation of the voiceband channel. This way it is possible to make a considerable reduction in the number of terms needed for the nonlinear cancellation filter (NLCF). The performance of the technique is tested by simulations of nonlinear voiceband communication systems. The results demonstrate that the NLCF can be considered a practical alternative for improvement of modern modem receivers

## 1. INTRODUCTION

It is well known that distortion in voiceband channels can have nonlinearities. Usually, the nonlinearities are caused by nonlinear circuits elements in telephone networks and have the forms of saturation. This source of nonlinearity is memoryless but in combination with linear filtering in the system, the overall response is nonlinear with memory, and, thus, nonlinear intersymbol interference (ISI) is induced. This ISI distortion has little or no effect on lower bit rate modems (2400 - 14.400 bps) but for higher bit rates (e.g. 28.800 bps and 33.600 bps) it can have an effect on the error rate. To be able to maintain these high bit rates for greater number of connections, than today's modems are capable of, an efficient method is needed to remove the nonlinear ISI. The objective of this paper is to cancel the nonlinear ISI with an algorithm which can be included in current modem designs without modifying the architecture of the receivers.

There are many possible strategies to combat nonlinear ISI. Most of them use either equalization or cancellation [1], [2], [3], [8]. Biglieri *et al.* [1] proposed two cancellation techniques. One can be included in present-day modems, and the other requires a change in the receiver architecture. Serfaty *et al.* [8] also proposed cancellation techniques that can be used in existing modems. The drawback of all these techniques is that they require an enormous number of nonlinear terms for both a moderate length of memory and degree of nonlinearity. Also, most of them use orthogonalization in every adapting step which makes the computational burden too heavy for present-day modems.

Here, a cancellation algorithm which uses Fast Orthogonal Search (FOS) [4], [5] to construct a truncated Volterra filter during start-up is investigated. The FOS selects which terms in the Volterra series should be used for the filter. The terms are selected from a predetermined set of candidates that have been chosen by examination of the theoretical response of the communication system which is modeled as a linear nonlinear (LNL) system. This allows us to reduce the number of possible candidate terms and make a considerable saving in computational cost. The FOS also finds the initial values of the coefficients of the filter. This way, the structure of the filter

is designed for the channel where it is used to compensate for distortion. This is important since the channel and the nonlinear distortion vary from one connection to another. After the filter has been identified by the FOS, then it is possible to switch to a simple Least Mean Square (LMS) algorithm [7] to make the coefficients adaptive, so the nonlinear cancellation filter (NLCF) will be able to follow any slow changes in the channel distortion during one connection. The computational cost of the algorithm is highest during start-up when the FOS is selecting terms, but this cost is within reasonable limits. After the start up phase, the computational burden drops, how much depends on the adaptive scheme used. This is very convenient because during the start-up phase most modems have some computational power to spare, since they do not use coded signals, such as Trellis code, until after the start-up phase.

The paper is organized in the following way. First, a model of a nonlinear voiceband communication system is introduced. Section 3 gives a short description of the FOS algorithm. In Section 4 the NLCF is proposed and simulation results are discussed in Section 5. Finally, concluding remarks are given in Section 6.

## 2. MODEL OF A NONLINEAR VOICEBAND COMMUNICATION SYSTEM

The model used in this paper simulates a modem transmitter, a nonlinear voiceband channel and a modem receiver. The model is shown in Figure 1, the following description refers to that picture. Box 1 shows the transmitter part of the modem. The data signal  $A_n$  is complex valued,  $A_n = a_n + jb_n$ , where  $a_n$  and  $b_n$  are two independent uniformly distributed random variables which can take on the discrete levels  $\{-31, -29, \dots, -1, 1, \dots, 29, 31\}$  for QAM1024 modulation (the number of points in the signal constellation set for V.34 and V.34+ are 960 and 1664 respectively). The data symbol rate was set to be 2400 symbol/sec. The data were upsampled by four to make room for the modulation. The LP1 is a pulse shaping FIR filter with 64 taps. The carrier frequency was  $f_c = 1700\text{Hz}$ . The second box in Figure 1 models the nonlinear voiceband channel. Note that before the nonlinear distortion is added, the sample rate is doubled to avoid the occurrence of aliasing. LP2 and LP3 are 16 tap FIR filters used in the upsampling and downsampling procedure. Only third order nonlinear distortion is included. The coefficient  $\beta$  controls the third order harmonic content. The nonlinear distortion is subtracted from the original signal to simulate saturation. After the nonlinear distortion has been included, the sample rate is lowered by a factor two. The linear part of the channel is implemented as a FIR filter. Box 3 in Figure 1 shows the receiver part of the modem. First the signal is demodulated, then it is sent through a matched filter LP4, identical to LP1. Thereafter, the signal is downsampled and fed into a  $\frac{T}{2}$  fractional spaced linear equalizer (LE). From the above it can be seen that the system at hand is an LNL system. We shall now derive the response of such a system to a QAM modulated signal. For our application it is convenient to describe the QAM signal as

$$x(t) = \cos(\omega_0 t) \sum_n a_n g(t - nT)$$

<sup>1</sup>Supported in part by the Icelandic Research Council (RANNIS) and Rockwell International Corporation.

$$+ \sin(\omega_0 t) \sum_n b_n g(t - nT) \quad (1)$$

where  $\omega_0$  is the angular carrier frequency,  $\frac{1}{T}$  is the symbol rate and  $g(t)$  is the transmitter filter (LP1). The overall response of the LNL system in Figure 1 is

$$\begin{aligned} y(t) = & \cos(\omega_0 t) \left[ \sum_k a_{k-d} H(t - kT) \right. \\ & + \beta \sum_k \sum_l \sum_m a_{k-d} a_{l-d} a_{m-d} G_1(t - kT, t - lT, t - mT) \\ & + \beta \sum_k \sum_l \sum_m a_{k-d} b_{l-d} b_{m-d} G_2(t - kT, t - lT, t - mT) \\ & + \sin(\omega_0 t) \left[ \sum_k b_{k-d} H(t - kT) \right. \\ & + \beta \sum_k \sum_l \sum_m b_{k-d} b_{l-d} b_{m-d} G_3(t - kT, t - lT, t - mT) \\ & + \beta \sum_k \sum_l \sum_m b_{k-d} a_{l-d} a_{m-d} G_4(t - kT, t - lT, t - mT) \\ & \left. \cos(3\omega_0 t) \left[ \beta \sum_k \sum_l \sum_m a_{k-d} a_{l-d} a_{m-d} \times \right. \right. \\ & G_5(t - kT, t - lT, t - mT) \\ & + \beta \sum_k \sum_l \sum_m a_{k-d} b_{l-d} b_{m-d} G_6(t - kT, t - lT, t - mT) \\ & + \sin(3\omega_0 t) \left[ \beta \sum_k \sum_l \sum_m b_{k-d} b_{l-d} b_{m-d} \times \right. \\ & G_7(t - kT, t - lT, t - mT) \\ & + \beta \sum_k \sum_l \sum_m b_{k-d} a_{l-d} a_{m-d} G_8(t - kT, t - lT, t - mT) \\ & \left. \left. + n(t) \right] \right] \quad (2) \end{aligned}$$

where  $H(t)$  is the overall linear response of the system,  $G_1(t_1, t_2, t_3), \dots, G_8(t_1, t_2, t_3)$  define the cubic response of the system, and  $n(t)$  is Gaussian noise. The terms that are modulated with angular frequency  $3\omega_0$  are not expected to contribute much to the ISI because of lowpass filtering in the telephone networks. It can then be seen that in the baseband, the ISI can be expected to have the form,

$$e_a(t) \approx \sum_{k \neq d} a_{k-d} H(t) \quad (3)$$

$$\begin{aligned} & + \beta \sum_k \sum_l \sum_m a_{k-d} a_{l-d} a_{m-d} G_1(t - kT, t - lT, t - mT) \\ & + \beta \sum_k \sum_l \sum_m a_{k-d} b_{l-d} b_{m-d} G_2(t - kT, t - lT, t - mT) \\ e_b(t) \approx & \sum_{k \neq d} b_{k-d} H(t) \quad (4) \\ & + \beta \sum_k \sum_l \sum_m b_{k-d} b_{l-d} b_{m-d} G_3(t - kT, t - lT, t - mT) \\ & + \beta \sum_k \sum_l \sum_m b_{k-d} a_{l-d} a_{m-d} G_4(t - kT, t - lT, t - mT) \end{aligned}$$

where  $d$  is the overall delay of the system. Ideally, the NLCF should be able to cancel the errors given in (3) and (4). In the next section, the FOS algorithm, which is a fundamental part of the proposed NLCF, will be described.

### 3. THE FAST ORTHOGONAL SEARCH

This section contains a short review of the FOS algorithm. This algorithm was proposed by Korenberg [4], [5] and can be used in identification of nonlinear systems with truncated Volterra series. In the next section, it will be shown how the FOS can be used as a part of an algorithm that effectively determines an NLCF. The algorithm enables one to pick out the most effective terms in a Volterra series (effective in the sense that these terms contribute more than other terms to the reduction of the mean squared error (MSE)), and thus enable us to construct a compact model of the system.

A discrete time Volterra series, with memory of length  $R$  and nonlinearity of degree  $\eta$ , has the form

$$\begin{aligned} y_v(n) = & k_0 + \sum_{j_1=0}^{R-1} k_1(j_1) x(n - j_1) \\ & + \sum_{j_1=0}^{R-1} \sum_{j_2=0}^{R-1} k_2(j_1, j_2) x(n - j_1) x(n - j_2) + \dots \\ & + \sum_{j_1=0}^{R-1} \dots \sum_{j_\eta=0}^{R-1} k_\eta(j_1, \dots, j_\eta) x(n - j_1) \dots x(n - j_\eta) \quad (5) \end{aligned}$$

where  $x(n)$  and  $y_v(n)$  are the input and output signal, respectively, and  $k_0, \dots, k_\eta$  are the Volterra kernels. The Volterra series can be written in a more convenient way as

$$y_v(n) = \sum_{m=0}^{M-1} a_m p_m(n), \quad (6)$$

where,  $M$  is the total number of terms in the truncated Volterra series and  $a_m$  presents the Volterra kernels. The goal of the identification is to select a new basis, for the filter which is a subspace of the space spanned by  $p_m$ ,  $m = 0, \dots, M-1$ , based on the mean squared error criterion between  $y_v(n)$  and the desired output and thus reduce the number of coefficients in (6). This is done by constructing an orthogonal basis  $W_m$  from  $p_m$  such

$$y_v(n) = \sum_{m=0}^M g_m W_m(n). \quad (7)$$

It can now be shown [6] that the MSE between  $y_v(n)$  and a desired output  $y(n)$  is

$$\text{MSE} = \frac{1}{N} \sum_{n=0}^{N-1} y^2(n) - \frac{1}{N} \sum_{m=0}^{N-1} g_m^2 W_m^2(n). \quad (8)$$

The FOS selects, at each orthogonalization step of  $p_m$ , a new term, for the filter, which has the highest value of  $g_m^2 W_m^2$ , among all the possible candidates. This procedure is repeated until some predetermined maximum number of terms have been selected or until the MSE has gone below some predetermined threshold. The FOS uses "fast" Cholesky decomposition to orthogonalize  $p_m$ , and does not require an explicit calculation of  $g_i$  and  $W_i$ .

### 4. THE NONLINEAR CANCELLATION FILTER

Here a cancellation scheme rather than equalization is chosen in order to prevent the need to find an inverse model for the nonlinear channel. The reason being that the inverse (if it exists) can have nonlinearities of higher order than the original model [3]. Here the structure for the NLCF is similar to the structure proposed by Biglieri *et al.* [1]. This structure is shown in Figure 2 where the NLCF is cascaded to the output of the equalizer and uses decision feedback (DFNLCF). Figure 2 shows the cancellation filter for the real part of the data signal, a identical cancellation filter is used for the imaginary part. The main difference between our NLCF and other filters previously introduced in the literature is that here the inner-structure is not defined beforehand. Rather, the FOS algorithm is used to design the structure of the filter for every connection. Thus, a filter can be designed in which all of its terms play an active role in reducing the MSE at the output and prevent the problem of wasting computational power by including terms that have little or no effect on the systems performance.

The filter tries to model the ISI. Then the filter uses the model to predict the ISI and subtracts the prediction from the signal that is to be corrected. The DFNLCF uses precursor and postcursor information when predicting the ISI, therefore a delay line is included at the output of the LE to make the system causal. Tentative decisions are used for postcursor symbols. The number of the linear and third order terms in (5) are  $\binom{3+R-1}{3} + R$ . We see that the number of terms can be enormous for moderate length of memory. For example, the number of terms for memory length 3 and 5 (thus in our case  $R$  is 6 and 10 since we have two inputs for the Volterra filter) are 62 and 230 respectively. We would thus need 124 and 460 terms, respectively, since we use two filters. The linear terms are included in the cancellation filter so it will be able to correct for linear ISI that are still remaining in the signal since the operation of the linear equalizer is affected by the nonlinearity in the channel.

We propose a two step procedure to reduce the number of terms. First we use the results from (3) and (4). By using only

the terms in (3) and (4) the total number of terms in the previous example are reduced from 124 and 460 terms to 68 and 240 terms, respectively. By doing this we make a considerable saving in the computational power needed for the FOS which reduces the number of terms much further.

The FOS algorithm is not adaptive. However, it is necessary to make the NLCF adaptive to compensate for slow variations in the channels response. There are many possible ways to accomplish this [7]. It is important to note that the FOS finds the initial values of the filter coefficients and the adaptive algorithm has only to be able to follow the slow changes in the channel. This allows the use of simple algorithms like LMS or normalized LMS.

## 5. SIMULATION RESULTS

The QAM1024 system described in Section 2 was used for simulation. In all the simulation the linear equalizer was trained on the first  $2 \times 20000$  symbols and then its adaption was stopped, the nonlinear cancellation filter was then identified with FOS, where  $2 \times 2000$  training symbols were used. In the first simulation the SNR was held constant at about 41dB for all runs. The third order nonlinear distortion was then varied from 29.7dB to 37.3dB. The system was simulated until the number of errors had reached 500. In the first simulation we compared three NLCF. Filter one used all the possible linear and third order terms in a Volterra filter with memory length 3 and delay 1, i.e., it had  $2 \times 62 = 124$  terms. The memory length of filter two was also 3 and the delay was 1, but it used only the terms shown in (3) and (4) and linear terms, and had  $2 \times 34 = 68$  terms. The coefficients of filter one and two were identified using the FOS algorithm without any term selection. Filter three had memory length 5 and delay 2. The terms of filter three were chosen from the terms in (3), (4) and linear terms, i.e., it had  $2 \times 120 = 240$  candidate terms (a reduction from  $2 \times 230 = 460$  candidate terms). Then  $2 \times 34 = 68$  terms were selected from the candidate terms using the FOS algorithm. Figure 3 shows the results. The first line shows the probability of error for system with no NLCF. Line labeled 2 shows the performance of filter one, line labeled 3 shows the performance of filter two and line labeled 4 shows the performance of filter three. Figure 3 shows that filters one and two perform identically, which supports the approximations made in (3) and (4). Filter three achieves considerable better performance than filter two, with the same number of coefficients. This is due to the fact that filter three is able to include terms with greater lag than filter two. When the third harmonic distortion are 32.5dB and 34.5dB filter three reduces the effect of the nonlinear distortion by about 2.4dB and 2.1dB, respectively, while filter two reduces the nonlinear distortion by only about 1.7dB and 1.5dB, respectively.

In the second simulation third order nonlinear distortion was held constant at about 32.5dB and the SNR was varied from 30dB to 73dB. Simulations were made for filter two, three and four. Here filter four has memory length 3 and delay 1, and number of coefficients were reduced from  $2 \times 34 = 68$  to  $2 \times 6 = 12$  terms by FOS. Figure 4 shows the results. The line labeled 1 shows the probability of error for system with no NLCF, line labeled 2 shows the performance of filter four, line labeled 3 shows the performance of filter two and line labeled 4 shows the probability of error for filter three. Finally, line labeled 5 shows the performance of a system with no nonlinear distortion. We see from Figure 4 that filter three does not show a better performance than filters two and four until the nonlinear distortion is 4dB higher than the Gaussian noise in the system. It is likely that the reason for this is that the nonlinear ISI terms have less power as the lag grows and are therefore buried in noise when the SNR and the nonlinear distortion are similar in power. Which leads to a difficulty in estimating the value of those terms. This makes it hard or even impossible to completely cancel the effect of the nonlinear distortion.

In the last simulation only a system with filter three and four were simulated, but the number of terms in the filter one were varied from  $2 \times 1 = 2$  to  $2 \times 73 = 146$  and for filter four from  $2 \times 1 = 2$  to  $2 \times 33 = 66$ , in steps of size two terms. The

SNR and third order nonlinear distortion were held constant at about 41dB and 32.5dB respectively. We see from Figure 5 that there is no gain in including more than  $2 \times 41 = 82$  terms in filter three and if more terms are added then we get worse performance. Similar behavior is seen for filter four, there nothing is gained by including more than  $2 \times 9 = 18$  terms. The reason for this is probably that the nonlinear error does not lie in the space spanned by the rest of the terms. These term will therefore only try to model the noise in the training data. The simulation results suggest that only about 15% of the available candidate terms play an active role in reducing the MSE.

## 6. CONCLUSION

An algorithm for cancellation of nonlinear intersymbol interference (ISI) in voiceband communication systems has been investigated. The algorithm uses fast orthogonal search (FOS) to construct a nonlinear cancellation filter (NLCF) at the beginning of the connection, from a predetermined set of candidate terms that have been selected by theoretical investigation of the channels response. Then the algorithm can be made adaptive by the use of the LMS algorithm or any other adaptive algorithm, since the resulting filter is linear in its coefficients. The algorithm is easy to implement and does not rely heavily on computational resources when implemented with lookup tables. Such implementation is possible when the same input data series is used for the identification with FOS. The algorithm can be looked at as an extension to modern modem receivers and it requires no change in the receiver architecture.

The NLCFs which were constructed with term selection outperform the other NLCF tested in the simulation. In voiceband communication channels the expected SNR can range from 34dB to 42dB. The simulation showed that the algorithm can perform well in this range and give a considerable gain in error probability. In the experiments it was shown that it is possible to reduce the number of terms in the Volterra filter with a given memory by more than 85%, without having effect on its performance.

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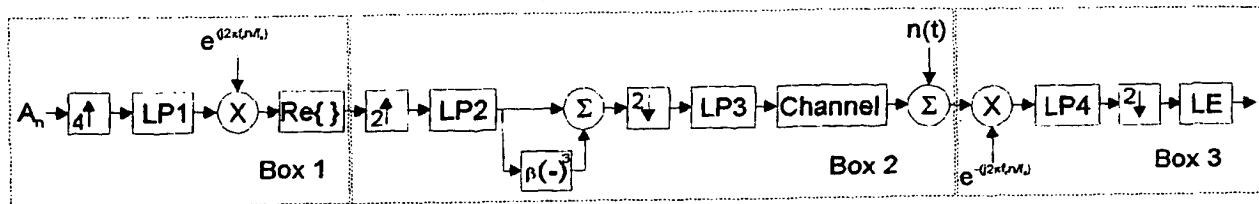


Figure 1. Blockdiagram of the communication system used in the simulations.

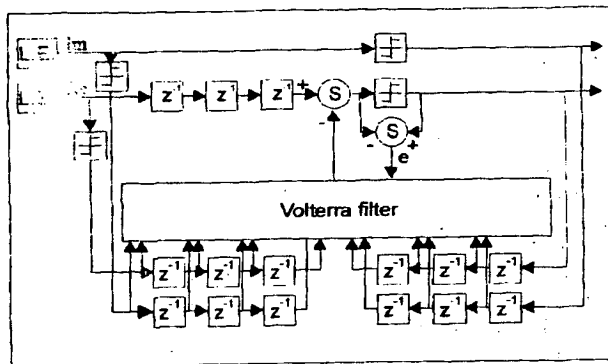


Figure 2. Blockdiagram of the nonlinear cancellation filter.

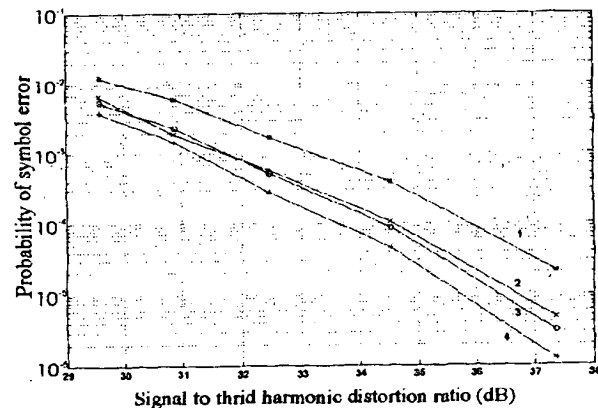


Figure 3. Results from the first simulation, see text.

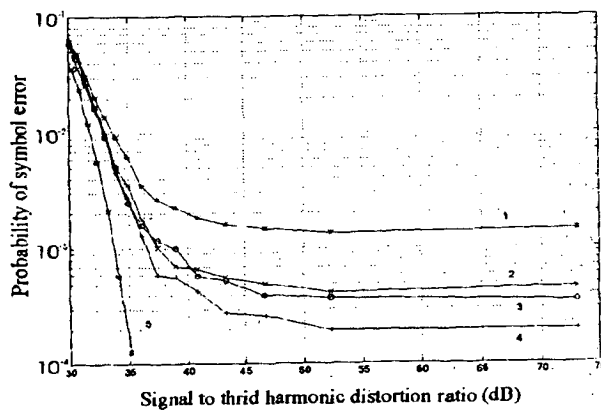


Figure 4. Results from the second simulation, see text.

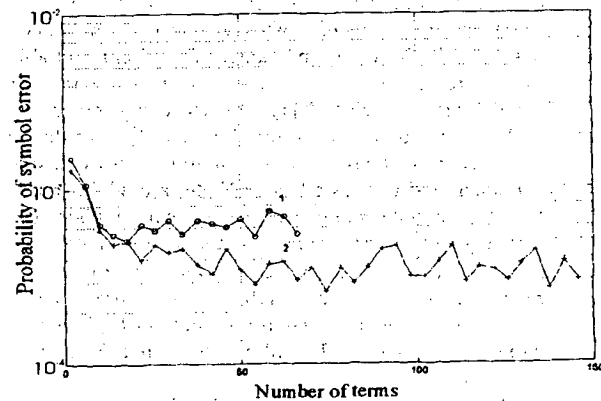


Figure 5. Results from the third simulation, see text.